# MATHEMATICAL PROGRAMMING BASIS FOR DECISION MAKING USING WEATHER AND CLIMATE INFORMATION FOR THE ENERGY SECTOR

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**Abstract.** Decision making (DM) problem is of great practical value in many areas of human activities. Most widely used DM methods are based on probabilistic approaches. Well-known Bayesian theorem for conditional probability density function (PDF) is a background for such techniques. It is due to some uncertainty in many parameters entered in any model described functioning of many real systems or objects. Uncertainty in our knowledge might be expressed in alternative form. I offer to employ appropriate confidential intervals for model parameters instead of relevant PDF. Thus one can formulate a prior uncertainty in model parameters by means of a set of linear constraints. Related cost or goal function should be defined at corresponding set of parameters. That leads us to statement of problem in terms of operational research or mathematical linear programming. It is more convenient to formulate such optimization problem for discreet or Boolean variables. Review of relevant problem statements and numerical techniques will be presented as well as several examples. The house heating and air condition optimal strategies responded to different IPCC climate change scenarios for some domains of Russia are considered. Evolving of climate and energy costs should be taken into account in building construction design. Optimal relationship between future expenses for house heating and costs of new house constructions including material costs and its amounts is a subject of discussion. In both considered tasks DM might be performed by means of the discreet optimization algorithms. If the DM variables are all required to be integers, then the problem is called an integer programming (IP). The "0-1" IP is the special case of integer programming where variables are required to be 0 or 1 (rather than arbitrary integers). The IP is a most convenient form for decision maker use. The "1" value means

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that a given scenarios is accepted, the "0" value means that a given scenarios is rejected. To illustrate suggested approach the "branch and bound" technique was implemented to seasonal surface atmosphere temperature ensemble predictions system (EPS) for northern parts of Russia. Aim of this illustrative research was to link the EPS output facility to requirements of particular users.

**Keywords:** Climate change; energy sector; decision making; operational research techniques; integer programming; branch and bound method

## 1. Introduction

An overview of key facts and societal challenges related to economic development, future energy demand and the impact that demand could have on the climate system might be found in some documents and papers (IEA, 2003, 2004, 2005; IPCC, 2007). These studies will help further elaborate a business response to the challenges identified in this paper, which will require additional research and consultation.

We cannot know exactly how the world will develop over the next half century, but the scenarios used here fit with the United Nations (UN) development goals of poverty reduction and improved living standards in the developing world. Achieving these goals will require an increase in energy consumption. Although we recognize that a range of human activities has an impact on greenhouse gas emissions and that many of these practices will have to change, the focus of this publication is on the world's use of energy, related impacts and on the decision making in energy sector to optimize relationships between expected benefits and losses (IPCC, 2007).

We have used existing data from the Intergovernmental Panel on Climate Change (IPCC), the International Energy Agency (IEA) and World Business Council for Sustainable Development (WBCSD) studies. We present it here in a simplified and condensed form to stimulate forward thinking and discussion around the issues facing us as we begin to deal with climate change.

Many paper space (BP, 2003; IEA, 2003, 2004, 2005; Meier, 2001; Von Moltke et al., 2004; WBCSD, 2004a, b, 2006) spent reviewing the different options for meeting green house gas (GHG) reduction targets, including energy supply options (carbon capture and sequestration from coal or natural gas, wind, solar, biofuels), energy efficiency improvements, alternative transportation technologies, and adaptation to climate change. In addition, many studies review the basic science behind climate change, including where

significant uncertainties remain: weather forecasting, inter-annual variability, trends in extreme weather events. House heating systems are the substantial energy savers when the DM is based on reliable forecast weather information concerned much shorter time scales (1–7 days).

Projections and examples based on particular global emission levels and eventual $CO_2$ concentrations in our atmosphere are only set out to illustrate the scale of the challenge. Another issue is to formulate optimization problem in energy sector to minimize GHG emission due to develop scenarios for better proportions between different energy producers and consumers.

## 2. Climate Change Impact on Energy Sector

The GHG that keeps our planet warm is getting thicker. This is because when we burn fossil fuels and cut down trees we add greenhouse gases to the atmosphere that trap the sun's heat. More heat means more unpredictable weather and other big changes for life on Earth. When we burn fossil fuels – oil, coal and natural gas – to make electricity, heat our homes and offices, cook, or power our cars, the main greenhouse gas carbon dioxide ($CO_2$) is released. Once released $CO_2$ can stay in the atmosphere for up to 200 years, heating up the planet.

The latest science from the Intergovernmental Panel on Climate Change (IPCC) – an independent global climate body – says that most of the warming in the past 50 years has been caused by humans. It predicts a temperature rise of up to 6°C by the end of the century. This means dangerous climate change, which we won't be able to avoid unless we act soon. The Kyoto Protocol, which became law in 2005, sets limits on the emissions of greenhouse gases from rich, developed countries like the UK. Many countries have signed up to the Protocol, but not the world's biggest emitter, the US, China, India.

The global economy is dependent upon oil and other fossil fuels, and this dependency (of producers and consumers alike) is fed through international trade. Increasing energy consumption, driven particularly by the rapid growth of emerging economies such as China and India, as well as volatile and rising oil prices and growing concerns over energy security are forcing a re-alignment of the global energy sector. Patterns of investment and technology flows in this sector are shifting in response to the scarcity of reserves, fostering the emergence of new actors and new strategies in the energy business.

The International Energy Agency (IEA) projects that energy demand and prices will continue to soar, with the world set to use 60% more energy in 2030 than at present. Renewable energies such as solar, wind, geothermal and modern biomass are on the rise, with wind power being the fastest

growing energy source in the world. However, incentives and investments in renewable energy sources continue to be insufficient to forge a fundamental overhaul of the energy sector.

The prevailing high oil prices encourage non-OPEC production of conventional and non-conventional oil. High oil prices are also likely to encourage the implementation of policies that reduce air pollution and GHG emissions enhance energy security. The higher the price of oil, the greater likelihood that alternative energy technology development will be promoted. Some renewable energy technologies are close to becoming commercially viable, while others occupy niche positions. Although development of renewable energy technologies on a wide-spread commercial basis is not expected in the short-term, high oil prices and technological developments will increase the opportunities for such energy sources.

IPCC's GHG emission scenarios are well known (IPCC, 2007). But the IEA and WBCSD provided more realistic ones due to more experience, deep linkages to industry and competence of their activity for many years (IEA, 2004). Moreover, IEA scenarios are more deeply substantiated by relevant economy research. In contrast IPCC scenarios are based on physical models and provide future evolving not only air temperature, but also some other variables (humidity, precipitation, wind velocity), which are important in more perfect economical models. Pathways to 2050: Energy and Climate Change (WBCSD, 2006) builds on the WBCSD's 2004 Facts and Trends to 2050: Energy and Climate Change (WBCSD, 2004a, b) and provides a more detailed overview of potential pathways to reducing $CO_2$ emissions.

The pathways shown illustrate the scale and complexity of the change needed, as well as the progress that has to be made through to 2050. Our "checkpoint" in 2025 gives a measure of this progress and demonstrates the urgency to act early to shift to a sustainable emissions trajectory. The WBCSD's principal sources of data for 2002 and 2025 references have been the IEA *World Energy Outlook* (IEA, 2004). While the WBCSD's projections were made in *Pathways to 2050* also build on the findings of *Facts and Trends* (WBCSD, 2004a*)*. Projections for energy consumption by sectors are presented at Figure 1 (energy units – ExaJoules [EJ], one ExaJoule is 278 billion kWh).

There are there main consumers: buildings (house heating and cooling), transport and industry. Transport is expected to follow a sustainable development scenario (at quasi-constant emission level), while two others will enhance energy consumption rate.

Technical progress in energy sectors provides some decreasing in $CO_2$ emission per unit of produced energy (IEA, 2003, 2005). From 1990 to 2002, economic efficiency improved by 1.4% per year, with only a small improvement in carbon intensity. If the goal is to stay on the indicated

pathway to 2050, maintaining this slow decarbonization trend through to 2050 would require a 78% improvement in energy efficiency across the economy, or an improvement of over 3% per year (WBCSD, 2006).

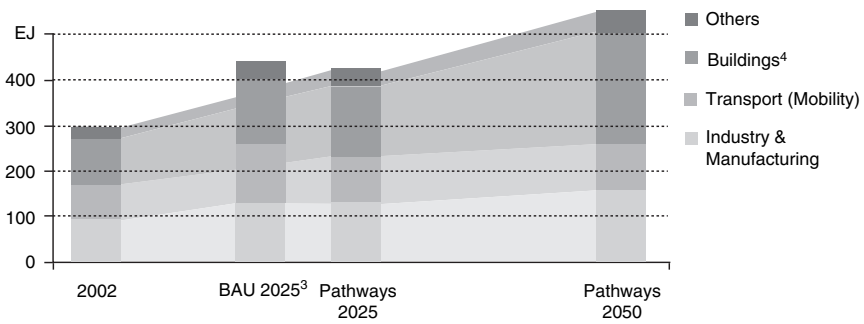


*Figure 1.* Energy consumption (in ExaJoules (EJ)) by sectors (WBCSD, 2006). (BAU 2025 – "Business As Usual" in 2025).

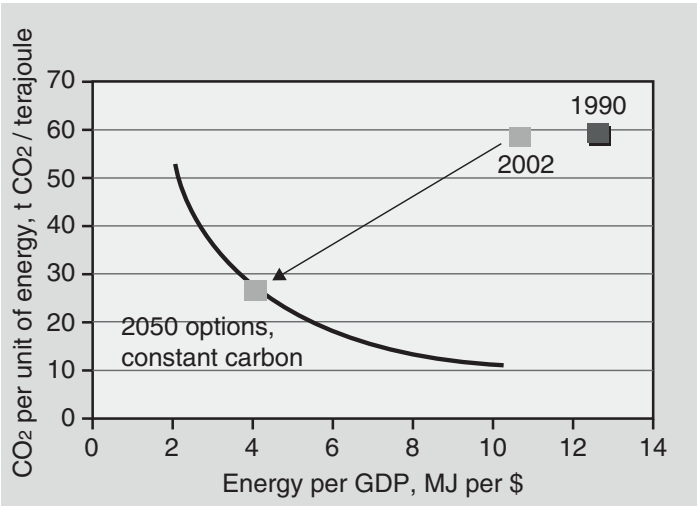


*Figure 2.* Relationship between carbon emission and produced energy amount for 2050 (WBCSD, 2006), values for 1990 and 2002 presented by single aside points.

Alternatively, with almost no improvement in energy efficiency, energy decarbonization must be some 80%, meaning an almost total renewable / nuclear / carbon sequestration based economy. Tackling both energy efficiency and carbon intensity is necessary. To 2050, an energy efficiency improvement of about 2% per year is assumed for North America, which in

turn requires a 1.4% per year reduction in carbon intensity (see Figure 2). GDP – Gross Domestic Product, a measure of the size of the economy; MegaJoules – MJ (A MegaJoule is one million Joules).

To handle with the decision making (DM) one needs some projections of major resource and economic variables. Primary energy (the total energy available from our natural resources, such as renewable, uranium, coal, oil and natural gas, assuming 100% efficient use of those resources) demand should be close to flat after 2025 (Figure 3), while maintaining economic growth (GDP per capita) and catering for growing population (WBCSD, 2006). GtC – carbon Giga Ton (A gigaton is equivalent to one billion tons).

Having in disposal above projections for carbon emission, GDP per capita, and primary energy along with energy distribution between different sectors (building, transport and industry) we are ready to formulate DM problem for optimal scenario to shear energy resources between its sectors.

We will consider mathematical statements of DM problem in next sections. Illustrative examples will be presented in Sections 5 and 6.

## 3. Decision Making Based on the Mathematical Programming Tools

Decision theory (DT) is theory about decisions. The subject is not a very unified one. To the contrary, there are many different ways to theorize about decisions, and therefore also many different research traditions. This text attempts to reflect some of the diversity of the subject. Its emphasis lies on the mathematical aspects of decision theory.

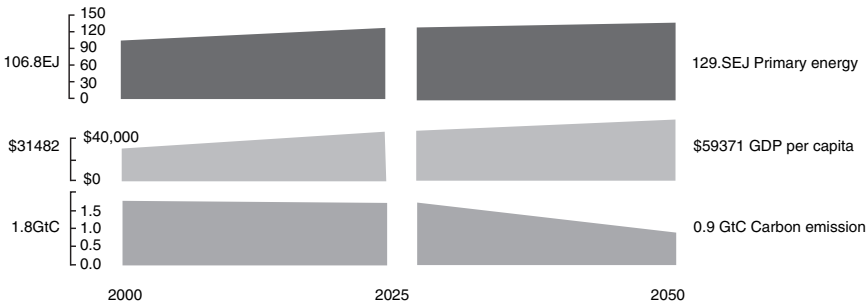


*Figure 3.* Projected changes in: (**a**) primary energy, (**b**) GDP per capita, (**c**) carbon emission for 2000–2050 (WBCSD, 2006).

Decision-theory focuses on how we use our freedom. In the situations treated by decision theorists, there are options to choose between, and we choose in a non-random way. Our choices, in these situations, are goal-directed activities. Hence, decision theory is concerned with *goal-directed behaviour in the presence of option*s.

We do not decide continuously. In the history of almost any activity, there are periods in which most of the decision-making is made, and other periods in which most of the implementation takes place. Decision-theory tries to throw light, in various ways, on the former type of period.

Decision makers divide the decision process into the following five steps:

- Identification of the problem
- Obtaining necessary information
- Production of possible solutions
- Evaluation of such solutions
- Selection of a strategy for performance

A set of above issues are *all sequential* in the sense that they divide decision processes into parts that always come in the same order or sequence. This approach might be criticized in such a way that the decision process can, in a general fashion, be divided into consecutive stages. Some empirical material indicates that the "stages" are performed in parallel rather than in sequence. A more realistic model should allow the various parts of the decision process to come in different order in different decisions.

Bayesian decision theory and relevant algorithms are the most popular tools for the DM (Raftery, 1996a, b). But there is very important disadvantage. One has to introduce a prior probability distribution for a set of the parameters to be estimated (Weakliem, 1999). Classical decision theory, namely Wald's maximin paradigm, is another direction in studies in the DM (Ben-Tal et al., 2006; Kouvelis and Yu, 1997). This general approach to optimization permitted to develop the *mathematical programming* (MP) models (Ecker and Kupferschmid, 1988; Thie, 1988; Kouvelis and Yu, 1997).

### 3.1. MATHEMATICAL PROGRAMMING

From an analytical perspective (Dantzig, 1949; Kantorovich, 1939, 1966), a mathematical program tries to identify an extreme (i.e., minimum or maximum) point of a function $\mathbf{f}(x_1, x_2, ..., x_n)$, which furthermore satisfies a set of constraints, e.g. $\mathbf{g}(x_1, x_2, ..., x_n) \geq \mathbf{b}$. Linear programming is the specialization of mathematical programming to the case where both, function f – to be called the objective function – and the problem constraints are linear.

From an applications perspective, mathematical (and therefore, linear) programming is an optimization tool, which allows the rationalization of many managerial and/or technological decisions required by contemporary

techno-socio-economic applications. An important factor for the applicability of the mathematical programming methodology in various application contexts is the computational tractability of the resulting analytical models. Under the advent of modern computing technology, this tractability requirement translates to the existence of effective and efficient algorithmic procedures able to provide a systematic and fast solution to these models. For Linear Programming problems, the Simplex algorithm, discussed later in the text, provides a powerful computational tool, able to provide fast solutions to very large-scale applications, sometimes including hundreds of thousands of variables (i.e., decision factors). In fact, the Simplex algorithm was one of the first *Mathematical Programming* algorithms to be developed (Dantzig, 1949), and its subsequent successful implementation in a series of applications significantly contributed to the acceptance of the broader field of Operations Research as a scientific approach to decision making.

## 3.2. LINEAR PROGRAMMING

*A Linear Programming* (LP) problem is a special case of a *Mathematical Programming* problem (Dantzig, 1949; Gass, 1958; Kantorovich, 1939, 1966).

### 3.2.1. *Illustrative Example*

Let us consider a simple example of the MP problem formulation (Kantorovich, 1966). Assume that a company produces two types of products $P_1$ and $P_2$. Production of these products is supported by two workstations $W_1$ and $W_2$, with each station visited by both product types. If workstation $W_1$ is dedicated completely to the production of product type $P_1$, it can process 40 units per day, while if it is dedicated to the production of product $P_2$, it can process 60 units per day. Similarly, workstation $W_2$ can produce daily 50 units of product $P_1$ and 50 units of product $P_2$, assuming that it is dedicated completely to the production of the corresponding product. If the company's profit by disposing one unit of product $P_1$ is \$ 200 and that of disposing one unit of $P_2$ is \$ 400, and assuming that the company can dispose its entire production, how many units of each product should the company produce on a daily basis to maximize its profit?

First notice that this problem is an optimization problem. Our objective is to maximize the company's profit, which under the problem assumptions, is equivalent to maximizing the company's daily profit. Furthermore, we are going to maximize the company profit by adjusting the levels of the daily production for the two items $P_1$ and $P_2$. Therefore, these daily production levels are the control/decision factors, the values of which we are called to determine. In the analytical formulation of the problem the role of these factors is captured by modelling them as the problem *decision variables*:

- $X_1$ := number of units of product $P_1$ to be produced daily
- $X_2$ := number of units of product $P_2$ to be produced daily

In the light of the above discussion, the problem objective can be expressed analytically as:

$$f(X_1, X_2) = 200X_1 + 400X_2 \tag{1}$$

Equation (1) will be called the objective function of the problem, and the coefficients 200 and 400, which multiply the decision variables in it, will be called the objective function coefficients.

Furthermore, any *decision* regarding the daily production levels for items $P_1$ and $P_2$ in order to be realizable in the company's operation context must observe the production capacity of the two workstations $W_1$ and $W_2$. Hence, our next step in the problem formulation seeks to introduce these technological constraints in it. Let's focus first on the constraint, which expresses the finite production capacity of workstation $W_1$. Regarding this constraint, we know that 1 day's work dedicated to the production of item $P_1$ can result in 40 units of that item, while the same period dedicated to the production of item $P_2$ will provide 60 units of it. Assuming that production of one unit of product type $P_i (i = 1,2)$, requires a constant amount of processing time $T_{1i} (i = 1,2)$ at workstation $W_1$, it follows that: $T_{11} = \frac{1}{40}$ and $T_{12} = \frac{1}{60}$. Under the further assumption that the combined production of both items has no side-effects, i.e., does not impose any additional requirements for production capacity of workstation (e.g., zero set-up times), the total capacity (in terms of time length) required for producing $X_1$ units of product $P_1$ and $X_2$ units of product $P_2$ is equal to:

$$\frac{1}{40}X_1 + \frac{1}{60}X_2 .$$

Hence, the technological constraint imposing the condition that our total daily processing requirements for workstation $W_1$ should not exceed its production capacity, is analytically expressed by:

$$\frac{1}{40}X_1 + \frac{1}{60}X_2 \leq 1 \tag{2}$$

Note that in equation (2) time is measured in days. Following the same line of reasoning (and under similar assumptions), the constraint expressing the finite processing capacity of workstation is given by:

$$\frac{1}{50}X_1 + \frac{1}{50}X_2 \le 1 \tag{3}$$

Constraints (2) and (3) are known as the technological constraints of the problem. In particular, the coefficients of the variables $X_i (i = 1,2)$ in them, $\frac{1}{T_{ij}} (i = 1,2)$, are known as the technological coefficients of the problem formulation, while the values on the right-hand-side of the two inequalities define the right-hand side vector of the constraints.

Finally, to the above constraints we must add the requirement that any permissible value for variables $X_i (i = 1,2)$ must be nonnegative since these values express production levels. These constraints are known as the variable sign restrictions. Combining equations (1)–(3), the analytical formulation of our problem is as follows:

$$\max f(X_1, X_2) = 200X_1 + 400X_2 \tag{4}$$

$$\frac{1}{40}X_1 + \frac{1}{60}X_2 \le 1$$

$$\frac{1}{50}X_1 + \frac{1}{50}X_2 \le 1$$

$$X_i \ge 0 (i = 1,2)$$

### 3.3. THE GENERAL LINEAR PROGRAMMING FORMULATION

Generalizing formulation (3), the general form for a Linear Programming problem is as follows (Thie, 1988):

Linear Objective Function (LOF) maximization:

$$\max\{f(X_1, X_2, ..., X_n)\} = \max\{\sum_{i=1}^{n} c_i X_i\} \tag{5}$$

under Linear Constraints (LC):

$$\sum_{j=1}^{n} a_{ij} X_j \begin{pmatrix} \le \\ or \\ = \\ or \\ \ge \end{pmatrix} b_i, (\mathrm{i} = 1, ..., m) \tag{6}$$

The LC (6) might be used in important particular cases, when variables signs are prescribed:

$$(X_j \geq 0) or (X_j \leq 0), (j = 1,...,n) \tag{7}$$

We conclude our discussion on the general LP formulation, by formally defining the solution search space and optimality. Specifically, we shall define as the feasible region of the LP of equations (4)–(6), the entire set of vectors $\mathbf{X} = (X_1,..., X_n)^T$ that satisfy the LC of (4) and the sign restrictions of (7). An optimal solution to the problem is any feasible vector that further satisfies the optimality requirements expressed by (5)–(7). Introducing integrality requirements for some of the variables in an LP formulation turns the problem to one belonging in the class of (Mixed) Integer Programming (MIP) or Integer Programming (IP) (Gomory, 1963; Korbut and Finkelstein, 1969).

### 3.4. GRAPHICAL LP'S INTERPRETATION

In this section, we consider a solution approach for LP problems, which is based on a geometrical representation of the feasible region and the objective function (Thie, 1988). In particular, the space to be considered is the n-dimens ional space with each dimension defined by one of the LP variables $(X_1, X_2)$. Thus we present an illustration for the two-variable case.

We start our investigation regarding the geometrical representation of 2-var linear constraints by considering first constraints of the *equality type*, i.e.,

$$a_1X_1 + a_2X_2 = b \tag{8}$$

It is a well-known result that, assuming $a_2 \neq 0$, this equation corresponds to a straight line with slope $s = -\frac{a_1}{a_2}$ and intercept $d = \frac{b}{a_2}$. In the special case where $a_2 = 0$ , the solution space (locus) of equation (8) is still a straight line perpendicular to the $X_1$ -axis, intersecting it at the point $(\frac{b}{a_1}, 0)$. Notice that the presence of an equality constraint restricts the dimensionality of the feasible solution space by one degree of freedom, i.e., it turns it from a planar area to a line segment.

Consider the *inequality constraint*:

$$a_1 X_1 + a_2 X_2 \begin{pmatrix} \leq \\ or \\ \geq \end{pmatrix} b \tag{9}$$

The solution space of this constraint is one of the closed *half-planes* defined by the equation (9). To show this, let us consider a point $(X_1, X_2)$, which satisfies equation (9) as equality, and another point $(X_1^{'}, X_2^{'})$ for which equation (9) is also valid. For any such pair of points, it holds that:

$$a_1 (X_1^{'} - X_1) + a_2 (X_2^{'} - X_2) \begin{pmatrix} \leq \\ or \\ \geq \end{pmatrix} 0 \tag{10}$$

Interpreting the left side of (10) as the inner (dot) product of the two vectors $\mathbf{a} = (a_1, a_2)^T$ and $\Delta\mathbf{X} = ((X_1^{'} - X_1), (X_2^{'} - X_2))^T$, and recognizing that $\mathbf{a}^T \cdot \Delta\mathbf{X} = |\mathbf{a}| \cdot |\Delta\mathbf{X}| \cos(\mathbf{a}, \Delta\mathbf{X})$, it follows that line $a_1 X_1 + a_2 X_2 = b$, itself, can be defined by point $(X_1, X_2)$ and the set of points $(X^{'}_1, X^{'}_2)$ such that vector **a** is at right angles with vector $\Delta\mathbf{X}$. Furthermore, the set of points $(X^{'}_1, X^{'}_2)$ that satisfy the > (<) part of equation (10) have the vector $\Delta\mathbf{X}$ forming an acute (obtuse) angle with vector **a**, and therefore, they are "above" ("below") the line. Hence, the set of points satisfying each of the two inequalities implied by equation (9) is given by one of the two half-planes the boundary of which is defined by the corresponding equality constraint. Figure 4 summarizes the above discussion.

An easy way to determine the half-plane depicting the solution space of a linear inequality, is to draw the line depicting the solution space of the corresponding equality constraint, and then test whether the point (0,0) satisfies the inequality. In case of a positive answer, the solution space is the half-space containing the origin, otherwise, it is the other one.

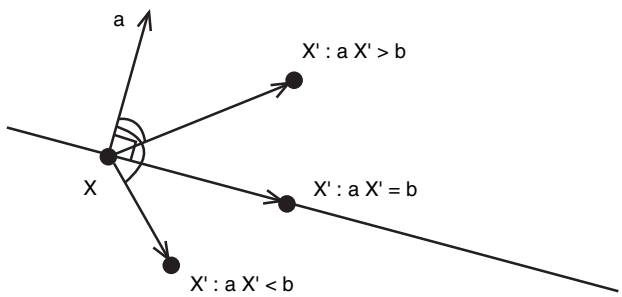


*Figure 4.* Half-planes: the feasible region of a linear inequality.

From the above discussion, it follows that the feasible region for the prototype LP of equation (4) is the shaded area in Figure 5:

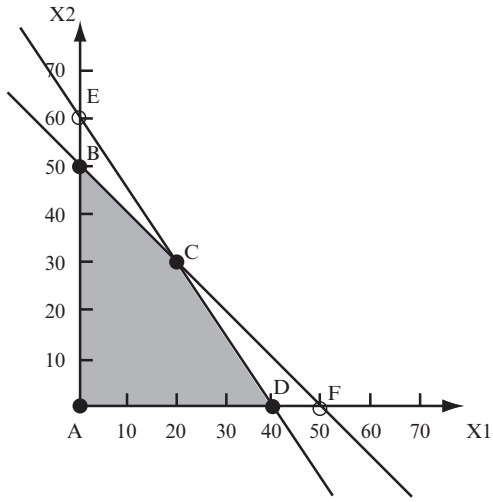


*Figure 5.* The feasible region of the example LP considered in 3.4.

Next step is a maximization (minimization) of objective function. The most typical way to represent a two-variable function $(a_1X_1 + a_2X_2)$ is to perceive it as a surface in an (orthogonal) three-dimensional space, where two of the dimensions correspond to the independent variables $X_1$and $X_2$, while the third dimension provides the function value for any pair $(X_1, X_2)$. However, in the context of our discussion, we are interested in expressing the information contained in the two-variable LP objective function $(a_1X_1 + a_2X_2)$ in the Cartesian plane defined by the two independent variables $X_1$ and $X_2$. For this purpose, we shall use the concept of contour plots. Contour plots depict a function by identifying the set of points $(X_1, X_2)$ that correspond to a constant value of the function $(a_1X_1 + a_2X_2) = a$, for any given range of $a$ 's. The plot obtained for any fixed value of $a$ is a contour of the function. Studying the structure of a contour is expected to identify some patterns that essentially depict some useful properties of the function. In the case of LP's, the linearity of the objective function implies that any contour of it will be of the type:

$$(a_1X_1 + a_2X_2) = a \tag{11}$$

i.e., a straight line. For a maximization (minimization) problem, this line will be called an isoprofit (isocost) line. Assuming that $c_2 \neq 0$, equation (11) can be rewritten as:

$$X_2 = -\frac{c_1}{c_2}X_1 + \frac{a}{c_2},$$

which implies that by changing the value of *a*, the resulting isoprofit/ isocost lines have constant slope and varying intercept, i.e., they are parallel to each other (which makes sense, since by the definition of this concept, isoprofit/isocost lines cannot intersect). Hence, if we continuously increase *a* from some initial value $a_o$, the corresponding isoprofit lines can be obtained by "sliding" the isprofit line corresponding to $(a_1X_1 + a_2X_2) = a_o$ parallel to itself, in the direction of increasing or decreasing intercepts, depending on whether $c_2$ is positive or negative.

The "sliding motion" suggests a way for identifying the optimal values for, let's say, a max LP problem. The underlying idea is to keep "sliding" the isoprofit line $(a_1X_1 + a_2X_2) = a_o$ in the direction of increasing *a* 's, until we cross the boundary of the LP feasible region. The implementation of this idea on the LP of equation (4) (see also Figure 5) is depicted in Figure 6. From Figure 6, it follows that the optimal daily production levels for the prototype LP are given by the coordinates of the point corresponding to the intersection of line $\frac{1}{50}X_1 + \frac{1}{50}X_2 = 0$ with the $X_2$-axis, i.e., $X_1^{opt} = 0; X_2^{opt} = 50$. The maximal daily profit is $200 \cdot 0 + 400 \cdot 50 = 20.000$ \$.

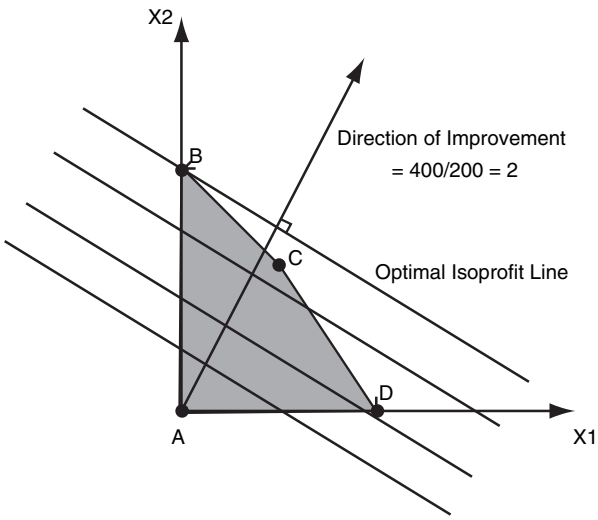


*Figure 6.* Graphical solution of the example LP (6).

Notice that the optimal point is one of the "corner" points of the feasible region depicted in Figure 3. Can you argue that for the geometry of the feasible region for 2-var LP's described above, if there is a bounded optimal solution, then there will be one which corresponds to one of the corner points? (This argument is developed for the broader context of n-var LP's in the next section.).

There are two fail options related to LP problem solution. First is absence of any solution, when feasible region is empty. Consider again the original example (6), modified by the additional requirements (imposed by the company's marketing department) that the daily production of product $P_1$ must be at least 30 units, and that of product $P_2$ should exceed 20 units. These requirements introduce two new constraints into the problem formulation, i.e., $X_1 \geq 30$; $X_2 \geq 20$. Attempting to plot the feasible region for this new problem, we get Figure 4, which indicates that there are no points on the $(X_1, X_2)$-plane that satisfy all constraints, and therefore our problem is infeasible (over-constrained).

Second particular option is an unbounded solution. In the LP's considered above, the feasible region (if not empty) was a bounded area of the $(X_1, X_2)$-plane. For this kind of problems it is obvious that all values of the LP objective function (and therefore the optimal) are bounded.

Summarizing the above discussion, we have shown that a 2-var LP can either

- Have a unique optimal solution which corresponds to a "corner" point of the feasible region, or
- Have many optimal solutions that correspond to an entire "edge" of the feasible region, or
- Be unbounded, or
- Be infeasible

## 4. Application to the Weather Ensemble Forecasting

### 4.1. MULTI-USER CONSORTIUM

Requirements for weather forecast products can vary significantly and are typically oriented to the needs of specific user groups. Nonetheless, in many respects the requirements are rather similar, such as a common need for information on basic variables such as temperature, humidity, and precipitation (mean, maximum, minimum). On other hand, it is hardly to imagine that every user could provide their own forecast product because of substantial costs of both inputs observing data and model development/maintenance.

In the case of specified forecast some additional observations might be required to increase prescribed reliability or probability. Therefore, it is more rational to select a set of few forecast models and observing systems, which respond to the right extent to optimal set of requirements generated by multi-user economical and mathematical model. Consortium of multi-user will get benefits of mathematically optimal decisions under minimal costs. User investments in a weather forecast system should be proportional to their expected benefits derived from the early warning of short-term weather fluctuations or extreme events. Under circumstances a consortium of multi-users approach would be more likely to derive benefits from the mathematically optimal decisions for minimum investment. Meteorological community is interested in such approach in order to reduce a number of observing programs and forecasting models. Latter might become a background to increase its efficiency (Pokrovsky, 2005, 2006, 2009).

### 4.2. ELEMENTARY STATEMENT OF THE PROBLEM

Let us assume that there are *n users* of climate forecasting data with their *n* benefits of early warning: $c_i$ (i = 1,.., *n*). These users are interested to forecast *m* specific *meteorological events* numerated as j = 1,…*m*. But usefulness of them are various and described by matrix of coefficients A={$a_{ij}$}. Each magnitude $a_{ij}$ can be considered as expenses of *i*-th user with account for *j*-th meteorological event delivered by some forecast model. *Minimum efficiency* for *i*-th user is bounded by value $b_i^{\min}$. Let us introduce *decision maker variable*:

$$x_i = \begin{Bmatrix} 1, if_i-th_user_adopts_forecast_data \\ 0, otherwise \end{Bmatrix}$$

Now we come to formulation of optimization problem for { $x_i$ }:

$$\max \sum_{i=1}^{n} c_i x_i \tag{12}$$

under constraints:

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i^{\min} \tag{13}$$

Another interpretation of coefficients and more complex method to derive them is possible. A generalization to the forecast multi-model case is evident.

### 4.3. ILLUSTRATIVE EXAMPLE

Let us consider a multi-user decision making to many meteorological events taken into account simultaneously. We used the ECMWF EPS prediction system for T850 anomaly, Europe, Jan–Feb, 1998 (see Figure 7) (details in Richardson, 2000) with n = 3 (number of users), m = 4 (number of meteorological events). Selected users respond to different values of the cost to loss ratio: C/L. The higher C/L – the less expected losses L under fixed protective expenses C, or highest protective costs under constant losses. Thus, the requirements of the user 3 are weaker than those of the user 1. On other hand, the EPS economical efficiencies presented in the raw of Table 1 are quite different for various meteorological events to be forecasted. Matrix of the EPS forecast relative economic values are presented in Table 1, minimal efficiencies for each user – in Table 2. We stated more minimal efficiency for the user 3 in order to compensate its maximal C/L value assumed lowest losses or highest protective costs. In the case of equal priorities of all users we come to the optimal solution $x_{opt}$ for (12) constrained by (13) (Table 3). This solution showed that the EPS forecasting system has prior importance for the user 2. Least contribution is related to the user 3. That might be explained by its highest C/L magnitudes. Let us now enhance a priory importance of the user 3 by changing values of target function (1) from c = (1, 1, 1) to c = (0.5, 0.5, 1). Even in this case the user 3 remains at second place after user 1. It is interesting to note that the output for the user 1 is an insensitive one with account to a priory weights.

TABLE 1. Matrix of constraints - $A = \{a_{ij}\}$.

| | Meteorological events: | | | |
|---|---|---|---|---|
| Users | (T<–8K) | (T<–4K) | (T>+4K) | (T>+8K) |
| 1 | 0.40 | 0.36 | 0 | 0 |
| 2 | 0.32 | 0.29 | 0.32 | 0.19 |
| 3 | 0.22 | 0.19 | 0.41 | 0.46 |

TABLE 2. Constraint vector of minimal efficiencies – $b_{min.}$

| Users | $b_i^{min}$ |
|---|---|
| 1 | 0.1 |
| 2 | 0.2 |
| 3 | 0.3 |

TABLE 3. Optimal decision "x" in the case of equitable users (c = (1, 1, 1), central column) and in the case of priority for the user n 3 (c = (0.5, 0.5, 1), right-hand column).

| Users | $x_{opt}$ | $x_{opt}$ |
|---|---|---|
| 1 | 2.31 | 2.26 |
| 2 | 5.21 | 0.36 |
| 3 | 0.85 | 1.99 |

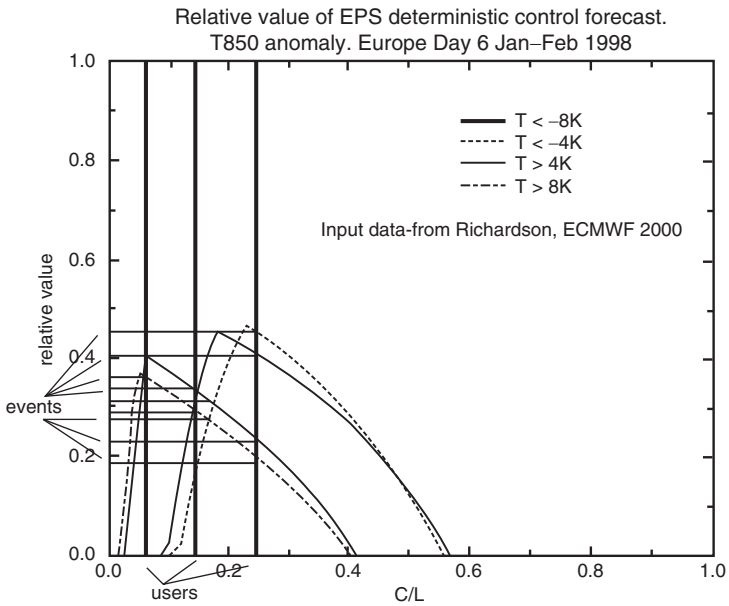


*Figure 7.* EPS forecast relative values responded to multi-user and multi-event case (Richardson, 2000).

## 5. Decision Making in Energy Sectors due to Climate Change Projections

### 5.1. IPCC GLOBAL SCENARIOS

The IPCC developed several scenarios of the GHG emissions scenarios to 2100, together with potential global temperature rise and associated climate impacts (IPCC, 2007). The IPCC scenarios are used here because of absence of similar detailed IEA and WBCSD scenarios concerned $CO_2$ reduction in an emission. A reduction of 6–7 Gt of carbon (22 Gt $CO_2$) emissions per year by 2050 compared to the A1B and B2 scenarios (Figure 8) would place us on a 550 ppm trajectory rather than 1,000 ppm $CO_2$, but a step-change evolution in our energy infrastructure would be required, utilizing resources

and technologies such as: (1) a further shift to natural gas nuclear energy; (2) renewables; (3) bio-products; (4) carbon capture and storage; (5) advanced vehicle technologies; and others.

A lower carbon world will require a marked shift in the energy/development relationship, such that similar development levels are achieved but with an average 30% less energy use. Both energy conservation through behaviour changes and energy efficiency through technologies play a role.

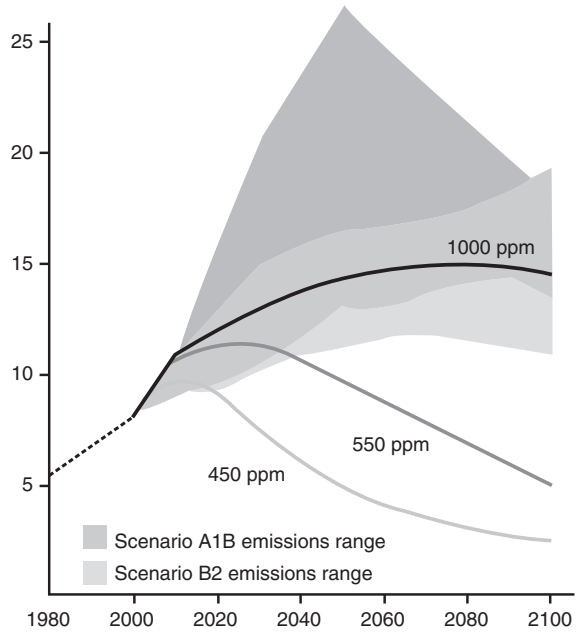


*Figure 8.* The GHG emissions scenarios to 2100 associated with potential global temperature rise and related climate impacts (IPCC, 2007).

Such a trend is a feature of the IPCC B1 storyline, which sees a future with a globally coherent approach to sustainable development. It describes a fast-changing and convergent world toward a service and information economy, with reductions in material intensity and the introduction of clean and resource efficient technologies.

## 5.2. STATEMENT OF REGIONAL OPTIMISATION STRATEGY

Above IPCC approach is relevant for global scale politics. Meanwhile, there is a question how to achieve prescribed emission limits in scale of each state, region and its economy. There are two statements aimed to restriction of GHG emission by means of optimal strategy implementation.

First one is related to minimization of total GHG emission. It might be formulated as following optimisation problem. We need to find a minimum of objective (cost) function:

$$\min\{\varepsilon\} = \min\{\sum_t \sum_i x_{t,i}\varepsilon(t, source(i))\} \tag{14}$$

under constraints:

$$0 < \sum_i x_{t,i}\varepsilon(t, source(i)) \le \delta(t) \tag{15}$$

Here $\varepsilon(t, source(i))$ is an emission of *i-th* regional source at time *t*. Admissible total regional emission at a given time *t* is $\delta(t)$. Objective function depends on decision making variables $x_{t,i}$, which indicate an individual weight of each emission source at given time *t*. Solution of (14)–(15) LP problem permits to suggest an optimal decision based on desirable relative (proportional) contribution of different sources in a given time slot in order to achieve minimum total emission under individual admissions responded to specific development conditions of *i-th* economic unit.

In many cases it is desirable to follow some prescribed emission scenarios developed for a given state, region or economy sector by international agreement(s). More specific statement of optimisation problem might be suggested in this case. Its main idea is a minimization of the deviation of the GHG emission from projected scenario. Object (cost) function in this case has a following expression:

$$\min\{|\varepsilon - \hat{\varepsilon}|\} = \min\{|\sum_t \sum_i x_{t,i}\varepsilon(t, source(i)) - \hat{\varepsilon}(t)|\} \tag{16}$$

under constraints:

$$0 < |\sum_i x_{t,i}\varepsilon(t, source(i)) - \hat{\varepsilon}(t)| \le \delta(t) \tag{17}$$

$\hat{\varepsilon}(t)$ is a prescribed emission trajectory for *i-th* source emission in (16) and (17). Admissible deviation from prescribed emission trajectory is described by $\delta(t)$ which is a function of *t*. Solution of (16)–(17) LP problem $\{x_{t,i}\}$ permits to suggest an optimal decision based on desirable relative (proportional) contribution of different sources in a given time slot in order to achieve minimum deviation of emission as function of time under individual admissions responded to specific development conditions of *i-th* economic unit. Next section suggest illustrative example for above approach.

### 5.3. ROAD TRANSPORT

Road transport GHG emissions contributed 1.5 Gt (gigaton) in 2000. This could rise to over 3.0 Gt by 2050 (WBCSD, 2004a) as the number of vehicles exceeds two billion. If all these vehicles increase efficiency level (e.g. using hybrid or advanced diesel technology) emission could be lower 1.0 Gt by 2050. If 800 million vehicles utilized new hydrogen technology (including novel cell technology) emission could also be lower 1.0 Gt by 2050.

Let us assume that a "step by step" increasing in efficiency of transport technologies is a most realistic scenario. That can lead to a smooth lowering in an emission rate by one car per year. This lowering might de different in various groups of cars. The vehicles provided by hybrid systems give most rapid decreasing in emission rates. Most promising is a hydrogen technology reducing GHG emission rate to zero. The hybrid technology is a more wide used now. It can lead to substantial contribution in total emission rate reduction in a group of vehicles (see Figure 9). Slower progress in emission rate decreasing is expected in a group of tracks with diesel engines.

Having in disposal a specific scenario for GHG emission rate evolving for next several decades in different group of transport one can use the DM algorithms developed in Section 4 to a problem of optimisation in development of various transport group to minimize total emission effect. In fact, let us introduce following notation:

$$a_{ij} = \varepsilon(t_j, source(i)) \tag{18}$$

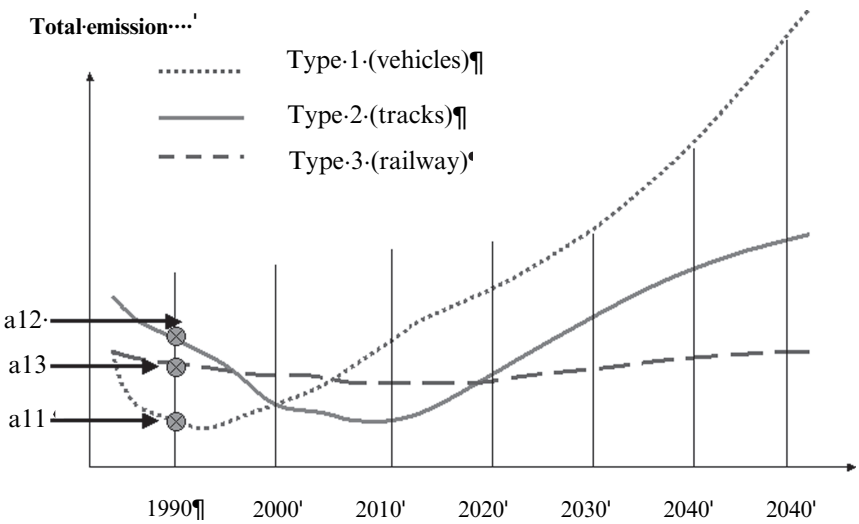


*Figure 9.* Emission rates in different group of transport: a scenario up to 2050.

for emissions considered in formulas (14)–(17) discussed in Section 5.2. Coefficients $a_{ij}$ entered in formula (6) (see Section 3) might be deduced from emission scenario presented in Figure 9. Further, decision maker can define the upper and lower admissible total emission rates per year and introduce these values as coefficients $b_i$ in (6). Function (5) might be determined on a base of regional conditions by decision maker.

Our nearest aim is to apply above technique to some climate change and energy supply scenarios discussed in this paper.

## 6. Conclusion

Approach based on MP and LP found a wide application area in many branches of economical sciences. It assisted in decision making related to multidimensional target function constrained by many linear cost restrictions. Major advantages of this approach with account to alternative Bayes techniques are related to: (1) a more simple interpretation of a prior uncertainty in search parameters by means of the confidential bands instead of the probability distributions function; (2) a more simple computations and more CPU time saving in a multidimensional case. This paper shows that similar problems arisen in many important practical areas might be efficiently solved by described approach.

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